A Framework of Residential Demand Aggregation With Financial Incentives

Qinran Hu, *Member, IEEE*, Fangxing Li, *Fellow, IEEE*, Xin Fang, *Student Member, IEEE*, and Linquan Bai, *Student Member, IEEE*

***Abstract*—The development of intelligent demand-side management with automatic control enables a large amount of residential demands to provide efficient demand-side ancillary services for load serving entities. In this paper, we introduce the concept of a comfort indicator, present an advanced reward system, and finally propose a framework for aggregating residential demands enrolled in incentive-based demand response (DR) programs. The proposed framework not only allocates load serving entities' demand reduction requests among residential appliances quickly and efficiently without affecting residents' comfort levels but also rewards residential consumers based on their actual participation. Also, since the framework is designed with the practical considerations of simplicity and efficiency, it can be utilized as a quick implementation for existing pilot development works. The effectiveness and merit of this framework are demonstrated and discussed in the comparison studies with conventional incentive-based DR.**

***Index Terms*—Demand response, power system economics, residential demand aggregation, smart grid, load serving entity.**

NOMENCLATURE

n	Number of households under one aggregator.
R_1	Level 1 reward rate, cents/(kW·5min).
R_2	Level 2 reward rate, cents/(kW·5min).
R_3	Level 3 reward rate, cents/(kW·5min).
$T_{RM,i}$	Room temperature of resident i, °F.
$T0_{RM,i}$	Initial room temperature of resident i, °F.
$T_{L,i}$	Low room temperature threshold of resident i, °F.
$T_{H,i}$	High room temperature threshold of resident i, °F.
PA_i	AC power rate of resident i, kW.
SA_i	Operating status of the AC of resident i, ON/OFF.
AE_i	Effect of the AC of resident i, °F/kW.
$LR_{RM,i}$	Room temperature loss rate of resident i.
$RWR_{A,i}$	AC reward rate for resident i, cents/(kW·5min).
$T_{T,i}$	EWH tank temperature of resident i, °F.
$T0_{T,i}$	EWH initial tank temperature of resident i, °F.
$T_{TL,i}$	Low tank temperature threshold of resident i, °F.
$T_{TH,i}$	High tank temperature threshold of resident i, °F.
PE_i	EWH power rate of resident i, kW.
SE_i	EWH operating status of resident i, ON/OFF.
E_i	Effect of the EWH for resident i, °F/kW.
$LR_{T,i}$	Tank temperature loss rate for resident i.
$RWR_{E,i}$	EWH reward rate for resident i, cents/kW·5min.
Com_i	If resident i compromises to the appliances operating beyond their comfort temperature ranges, YES/NO.
CI_i	Comfort indicator for resident i.
$TA_{RM,i}$	Ambient temperature for resident i.
$TA_{T,i}$	Ambient temperature for the EWH of resident i.
TDR	Total demand secluded to reduce, kW.
D	Amount of demand reduction required, kW.
δ	Parameter associated with demand reduction accuracy relaxation.
RW_i	Total financial rewards for resident i, \$.
w	Weight of comfort indicator.
μ_i, υ_i	Auxiliary binary variables for converting the optimization problem.

I. INTRODUCTION

DUE TO the integration of an increasing amount of distributed energy resources, power systems are inclined to have less conventional generators, which implies less generation reserve capability. Residential demand-side resources, which constitutes 38% of total electricity energy consumption in the U.S. [1], have the potential to be aggregated to facilitate power system operation.

Conventionally, load serving entities (LSEs) aggregate demand via price-based demand response (DR) programs such as time-of-use pricing, critical peak pricing, real-time pricing, and peak load reduction credits [2]–[9]. In recent years, due to the development of home energy management systems [10]–[12], LSEs start to conduct pilots of incentive-based DR programs on intelligent loads to explore the possibility of increasing their profits by aggregating residential demands [13]–[17]. However, the participants of these pilots report that their comfort levels were affected. Also, existing reward systems are not sufficiently attracting more DR program participants. For example, in the pilot study named "coolNYC," with a simple reward mechanism (\$5 per event), the program opt-out rate after a short-term trial is around 30% among the initial signups and the opt-out rate during an actual

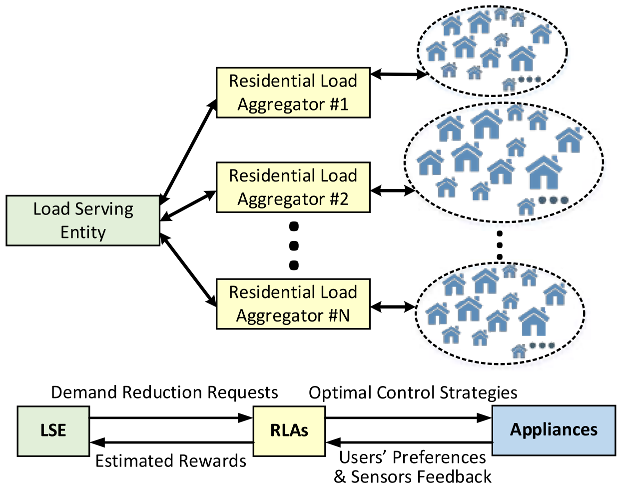


Fig. 1. Schematic flow chart of the framework.

DR request is even as high as 45%. Hence, this paper is intended to develop a comprehensive and efficient framework, which can be utilized as a quick implementation for existing pilots, for aggregating automated control enabled residential demands.

Various algorithms and techniques of aggregating residential demands have been discussed in the literature. In [18] and [19], the models to control the aggregated demand of air conditioners (ACs) by adjusting their temperature settings are proposed. Reference [20] considers residents' energy consumption habits and comfort preferences, and then proposes a method to characterize the availability of residential appliances in providing system reserve services. However, the previous studies seldom consider how to coordinate residents' energy consumption preferences and their comfort levels. Moreover, few articles present an advanced reward distribution system, which is essential, because it may directly affect the residents' participation levels.

To solve the issues as mentioned above, this paper proposes a framework of aggregating the residential demands enrolled in incentive-based DR programs for LSEs to provide demand-side ancillary service. Under this framework, LSEs are expected to control residential appliances based on residents' unique energy consumption preferences and impartially reward the participants.

In the proposed design, residential load aggregators (RLAs) serve as the agents who receive demand response requests (DRRs) from LSEs and real-time environmental parameters from every household as shown in Fig.1. Then, the RLAs generate the optimal operation strategy of appliances based on residents' preferences and send the optimized strategies to those appliances.

For DR program participants, this framework can 1) distribute financial rewards according to their quantified contributions in DR events, and 2) maintain their levels of in-home comfort based on their personal energy consumption preferences. For LSEs, this framework can 1) realize the DRR by controlling residents' appliances, and 2) minimize the total reward costs for performing DRRs. Hence, by benefiting both program participants and LESs, this framework is expected to attract more DR program participants and further stimulate the potential capability of controllable residential demand-side resources.

The rest of this paper is organized as follows. Section II presents the overall framework structure. Section III introduces the reward system of the proposed framework. Then, Section IV describes the proposed model, which is formulated as a mixed-integer quadratic program (MIQP) problem to minimize the total reward payment from LSEs to residents while maximizing the residents' comfort levels. The simulation results and numerical analysis of a ten-resident small system and a large-scale system are presented in Section V to demonstrate the effectiveness of the proposed framework. The summary and conclusion are presented in Section VI.

II. Overview of the Optimal Framework

According to several studies conducted by LSEs [21]–[26], ACs and electrical water heaters (EWHs) are critical in DR programs, because they are predominant inertia loads and able to provide fast responses with minimal impact to residents in a short period. Also, ACs and EWHs typically account for more than 50% of the residential peak demand [27]. Therefore, RLAs are expected to perform DRRs by controlling ACs and EWHs without affecting residents' comfort levels, while rewarding residents by quantifying the contributions they made.

There are several assumptions for the proposed framework: 1) ACs and EWHs have bi-directional communication with RLAs, which indicates that RLAs can obtain real-time room temperature from ACs, tank water temperature of EWHs, as well as their ON/OFF operating status; 2) the real-time ambient temperature is known to RLAs; 3) residents provide comfort temperature ranges of both indoor air and hot water to RLAs; and 4) residents have the choice to decide whether they are willing to sacrifice when RLAs have to adjust (lower) the comfort levels of some residents.

Fig. 2 is a schematic diagram of the information flow among LSE, RLA, and residents. When RLA receives a request from LSE notifying that there is demand D_r that needs to be reduced, this RLA considers residents' appliance profiles, their energy consumption preferences, real-time ambient temperature, and real-time indoor air temperature as well as water temperature. Based on this information, RLA performs optimization within a very short time. As a result, the framework achieves several tasks including: 1) generating and sending out optimal control instructions to residents' appliances, 2) providing the LSE with a cost-effective realization of the DRR with minimal reward costs, 3) recording the contributions that individual residents made for this DRR, and 4) distributing the financial rewards to the residents.

III. The Reward System

There are 55 utilities all over the U.S. offering incentive-based demand response programs to their residential customers. However, few existing programs provide the residents with an opportunity to customize their energy consumption

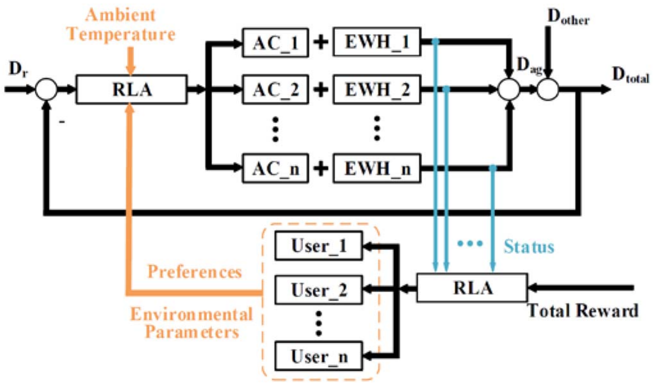


Fig. 2. Schematic diagram of the information flow.

TABLE I
VARIOUS REWARD RATES

Resident Type	*Rate*	*Symbol*	*Probability*
Compromise	Common	R_1	Common
	Higher	R_2	Occasional
Not Compromise	Common	R_1	Common
	Highest	R_3	Emergency

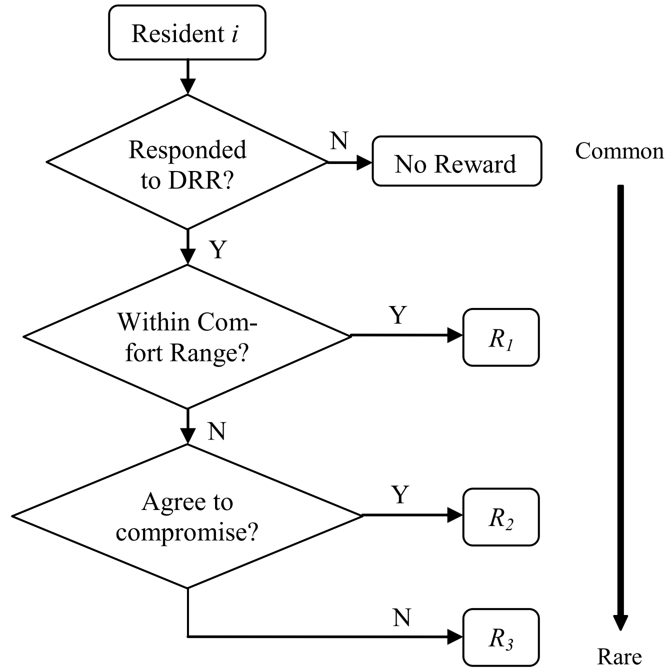


Fig. 3. The flow chart of determining the reward rate for resident i.

preferences. Further, most of the programs do not minimize the overall impact of DR events to residents' comfort levels. It needs to be noted that an appealing reward system is one of the vital factors to make such a framework feasible and then attract sufficient demand-side resources to provide ancillary services. Therefore, this section introduces the innovative reward system, which is implemented in the proposed framework.

A. Multilevel Reward Rates

This framework rewards DR program participants strategically. To get rewards, participants need to provide their energy consumption preferences including their comfort temperature setting ranges for EWH and AC units, and whether they are willing to compromise by turning off EWH and AC units when either the room or water tank temperatures go beyond their comfort temperature ranges. With large DRRs enforcing the RLA to work out a compromise with some residents' comfort, those residents will receive extra compensation, which means a higher reward rate. Moreover, if an emergency occurs, in order to maintain the stability of the power system, the LSE has to send a DRR with a tremendous amount to RLAs. Then, RLAs figure out that executing such DRR will have to make the appliances of residents, who claim not to compromise, operate beyond their comfort temperature ranges. In this case, those residents will get the highest reward rate.

The differences among various reward rates are shown in Table I. Taking AC units for example, the reward rates for resident i is determined based on the flow chart as shown in Fig. 3. Mathematically, the various reward rates can be expressed as (1). Since one of the objectives of the optimal framework is to minimize the total reward payment to perform certain DRR, naturally, the higher the reward level is, the less probability such situations happen. (The total reward payment minimization will be discussed in Section IV.) In order to achieve better effects of residential demand response, the values of R_1, R_2, and R_3 may vary from case to case, either from engineering experience or by analytical approaches such as game-theoretic study between LSEs and residents during the long-term planning process.

It should be noted that it is not necessarily a viable option for residents to intentionally choose "not to compromise" to gain more financial benefit with the expectation to receive the highest reward rate, R_3. The reason is that emergency cases may occur very rarely such that the customer may not have much chance to receive payment at the rate R_3 while losing more probable opportunities to receive rebates at R_2

$$RWR_{A,i} = \begin{cases} R_1, & if \ \ T_{L,i} \le T_{RM,i} \le T_{H,i} \\ R_2, & if \ \ T_{RM,i} \le T_{L,i}, \ \ Com_i = 1 \\ R_2, & if \ \ T_{H,i} \le T_{RM,i}, \ \ Com_i = 1 \\ R_3, & if \ \ T_{RM,i} \le T_{L,i}, \ \ Com_i = 0 \\ R_3, & if \ \ T_{H,i} \le T_{RM,i}, \ \ Com_i = 0. \end{cases} \tag{1}$$

B. Comfort Indicator

When allocating DRRs to appliances, the RLA always faces the issue of how to select the proper available appliances to participate in specific DRRs. Here, the proposed framework introduces the concept of a "Comfort Indicator" to solve this issue. Here, we may take resident i with an AC unit as an example. The "Comfort Indicator", CI_i, is defined by (2), and the mean value of the low and high threshold (user energy consumption preferences) of the comfort temperature range $\frac{T_{L,i}+T_{H,i}}{2}$, is assumed to be the ideal operating point. Then, CI_i stands for the distance between the present status and the ideal operating point. Therefore, the higher the CI_i value is the less comfortable the resident i feels. When the temperature goes

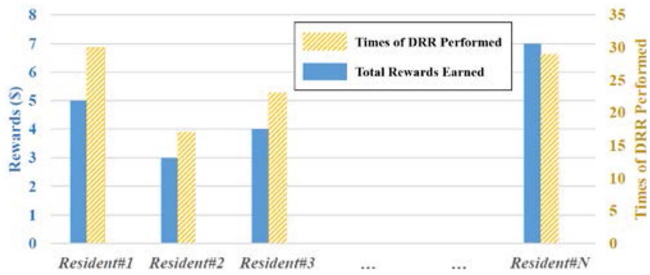


Fig. 4. The records of DRR participation history.

beyond the comfort temperature range, $CI_i > 1$.

$$CI_i = \left| \frac{2T_{RM,i} - T_{L,i} - T_{H,i}}{T_{H,i} - T_{L,i}} \right| \tag{2}$$

Substituting (2) into (1), the relationship between CI_i and the reward rates can be expressed in (3).

$$RWR_{A,i} = \begin{cases} R_1, & if \quad CI_i \leq 1 \\ R_2, & if \quad CI_i > 1 \quad \text{and} \quad Com_i = 1 \\ R_3, & if \quad CI_i > 1 \quad \text{and} \quad Com_i = 0 \end{cases} \tag{3}$$

To be fair for all the residents under the control of the same RLA, when the RLA receives a DRR, it should try to maintain a similar comfort margin for each resident in the controlled area while performing the demand reduction. This issue will be solved in Section IV, since overall comfort levels have been considered in the objective function of the optimization problem. However, there is still an issue among the residents with the same *CI* values. To address this problem, the RLA keeps a record of the DRR participation history for every resident. Thus, whenever the residents have the same *CI* values, the RLA will choose the one with a lower DRR contribution history to participate to maintain a fair and equal opportunity for all participants. For example, assume the contribution history for all the residents is as shown in Fig. 4. If resident #2 and #3 have the same *CI* value and either resident #2 or #3 has to turn off their AC unit for a while, resident #2 will be selected according to the aforementioned rule.

Furthermore, it needs to be noted that the record of DRR participation history and reward distribution results will be kept in RLAs' database. Those data will only be uploaded to the LSE every week or month, which can help relieve the stress for the LSE from having massive real-time bi-directional communication with tens of thousands of residents.

C. Discussion of Residents' Strategy

An effective reward system should be able to attract more participants into the DR programs and prevent any malicious manipulation. This reward system provides a platform where the residents can make a tradeoff between their comfort levels and financial rewards. By customizing their preferences, residents will be rewarded differently.

Here, a simple example of residents A, B, and C, each equipped with an AC unit under the same RLA, is used to perform a general analysis of the impact of different residents' preference settings to the financial reward they may obtain. Table II shows the preference settings of A, B, and C.

TABLE II
PREFERENCE SETTINGS FOR THREE RESIDENT EXAMPLE

Resident Name	*Comfort Temperature Range (°F)*	*Compromise?*
A	73-77	No
B	70-80	No
C	70-80	Yes

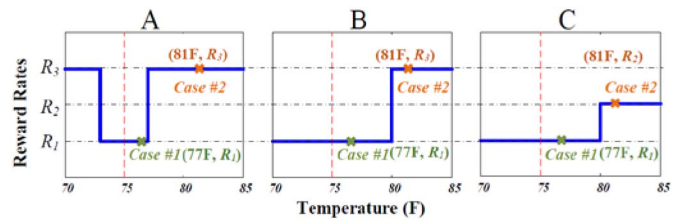


Fig. 5. Reward rate with predicted estimate room temperatures.

The comfort temperature ranges of B and C are broader than A's; and C chooses to compromise while both A and B choose not to.

Assume that today is a hot summer day. The residents A, B, and C have the same houses and AC units, and the present room temperatures are all at ideal operating point as mentioned in Section III-B (A: 75°F, B: 75°F, and C: 75°F). Fig. 5 shows the preference settings of the three residents, the blue curve represents the reward rates they will get with different estimated room temperatures during the demand reduction period, and the red dotted line is the initial room temperature before DRR.

Here, assume the RLA receives a DRR asking for a one-third reduction of total demands. Therefore, the RLA needs to turn off one of the ACs from these three residents. Here, we consider two cases, CASE#1 and CASE#2, in the discussion. The estimated room temperatures with executing this DRR for CASE#1 and CASE#2 are marked in Fig. 5 as green and orange crosses, respectively.

In CASE#1, because the houses, the AC units, and the initial room temperatures are identical, the estimated room temperature with executing this DRR is 77°F (marked as green) for all three residents. At this temperature, the reward rate is the same for all of them. However, due to the concern of $CI(CI_A = 1, CI_B = 0.4, CI_C = 0.4)$, resident A is excluded, while B and C share the same possibilities to turn off their ACs.

In CASE#2, the estimated room temperature with executing this DRR is 81°F (marked as orange) for all three residents. 81°F is beyond the comfort temperature ranges for A, B, and C ($CI_A, CI_B, CI_C > 1$). Due to the different preferences on "Compromise", the reward rate for C is lower than A and B. Therefore, C will be selected to turn off his/her AC.

This simple example clearly shows that resident C will most likely get the chance to perform a demand reduction and gain financial rewards, because C's settings indicate that she/he is willing to compromise her/his comfort level more than the others.

Please note that this example is only used here for a general demonstration of how the reward system works with different residents' preferences. The practical cases are inevitably much more complex due to the differences in houses, appliances' parameters, initial room temperatures, etc. The next section will provide the mathematical formulation of the proposed optimal framework with this reward system.

IV. Optimization Problem Formulation

The objective of the optimal framework is to realize cost-effective DRR while maintaining the comfort levels of residents. In the formulation of the model, there are several issues with the time length of DRR, temperature estimation, demand reduction accuracy, etc. This section will discuss these problems first and then formulate the complete optimization model as an MIQP problem which is solvable by available optimization software tools.

A. Time Length of DRR

A DRR contains two important factors: 1) total required demand reduction and 2) time length based on how long demand reduction should last. In order to prevent the discomfort caused by performing a single DRR with a long time period, the time duration of each DRR is set to five minutes, which means the long time period DRR can be treated as several continuous short ones.

There are several other advantages for dividing a long DRR into short time segments. This approach ensures a stable calculation time and makes the online optimization possible, because it keeps the size of the optimization problem unchanged. Also, at the beginning of each short DRR, sensors' feedbacks are collected by RLAs. This approach not only helps correct the errors in estimating room temperature but also eliminates the potential errors of water temperature caused by water usage.

Taking a one-hour long DRR with only ACs in winter as an example, the DRR will be divided into twelve five-minute DRRs. As shown in the schematic process chart in Fig. 6, after performing each of the short DRRs, the RLA receives the updated indoor air temperature data (green crosses) from each household, then performs next short DRR after 5 minutes. This approach maintains the residents' comfort levels during DR events, reduces calculation errors, and makes the online optimal DRR scheduling feasible.

B. Temperature Estimation

Temperature estimation is vital in this model, because it determines the reward rates for the residents.

As for estimating the water temperature in EWHs, the general model has been discussed in [28]–[30]. The discrete state dynamics EWH model is applied here, since the time length of each DRR is set to five minutes, which is a fixed duration. Hence, the model can be described by (4):

$$T_{T,i} = T0_{T,i} - LR_{T,i} \cdot \left(T0_{T,i} - TA_{T,i}\right) + E_i \cdot PE_i \cdot SE_i \quad (4)$$

As for estimating the indoor air temperature with AC units, the American Society of Heating, Refrigeration and Air Conditioning Engineers, Inc. has compiled modeling procedures in its fundamentals handbook [31]. The U.S. Department

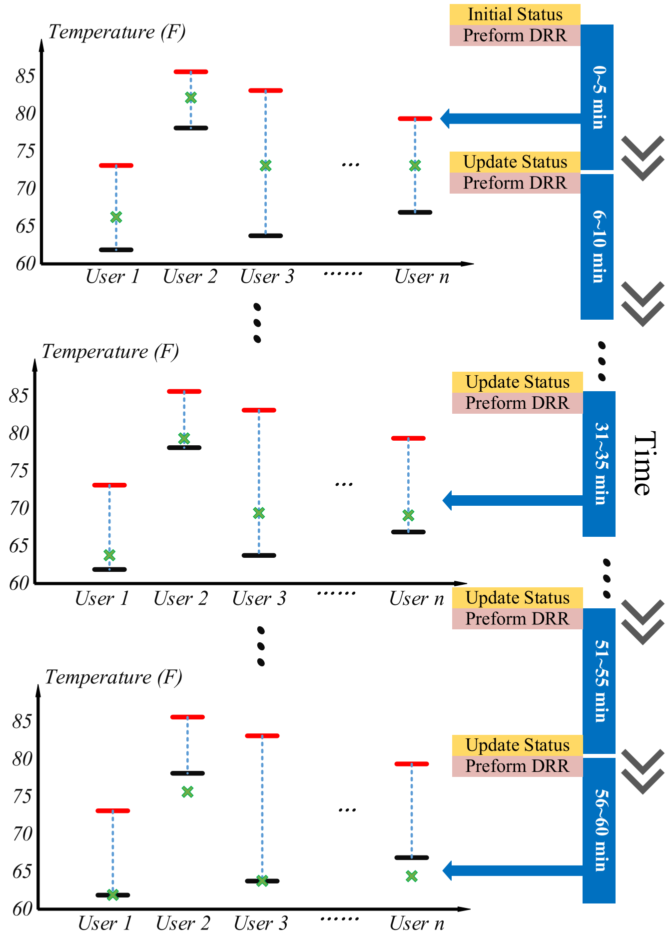


Fig. 6. The process chart for performing one hour length DRR.

of Energy has produced the Energy Plus program for computer simulation [32]. Also, the detailed model for simulating AC systems is given in [33] and [34]. According to these studies, an accurate model needs to consider many factors including weather, season, building thermal resistance, solar heating, the cooling effect of wind, shading, etc. Unlike EWH which has constant and accurate parameters, those parameters of AC are difficult to be precisely measured, since they are always time-varying with the operating status. If compared with the complex model, the simplified model, which is faster but less accurate, is better for the proposed framework, since the errors of predicting the temperature for only five minutes ahead are limited. The simplified model of estimating the indoor air temperature with ACs is as below.

$$T_{RM,i} = T0_{RM,i} - LR_{RM,i} \cdot \left(T0_{RM,i} - TA_{RM,i}\right) + AE_i \cdot PA_i \cdot SA_i \quad (5)$$

Instead of executing complicated setting adjustments, the ACs are simply controlled by ON/OFF. The control variable for ACs is the binary variable SA_i in (5).

It needs to be highlighted that the values of parameters $LR_{T,i}, E_i, LR_{RM,i}$, and AE_i are different for each resident. Since the RLA can receive the feedbacks of sensors, those parameters can be obtained through performing regression on the historical data for each resident in practical application.

However, due to the lack of historical data, those parameters are only set by assumptions in the numerical case studies in Section V.

C. Demand Reduction Accuracy Relaxation

The total demand can be reduced from n residents by executing the optimal control strategies over ACs and EWHs. This is expressed in (6) below.

$$TDR = \sum_{i=1}^{n} PA_i \cdot (1 - SA_i) + PE_i \cdot (1 - SE_i) \tag{6}$$

However, since SA_i and SE_i are both binary, TDR and demand D, which is the value requested to reduce, cannot be exactly the same, typically. It is also possible for D to go beyond the capability of the RLA. Therefore, the constraint of the amount of demand to be reduced needs to be relaxed according to the LSE requirement as expressed in (7).

$$(1 - \delta) \cdot D \leq TDR \leq (1 + \delta) \cdot D \tag{7}$$

The value of δ is set as 0.05 in the case studies to be discussed in Section V.

D. MIQP Problem Formulation

According to the discussion in Section III-A2, in summer time, the reward rates of ACs and EWHs are expressed by (8) and (9), respectively.

$$RWR_{A,i} = \begin{cases} R_1, & \text{if} \quad T_{RM,i} \leq T_{H,i} \\ R_2, & \text{if} \quad T_{H,i} < T_{RM,i} \text{ and } Com_i = 1 \\ R_3, & \text{if} \quad T_{H,i} < T_{RM,i} \text{ and } Com_i = 0 \end{cases} \tag{8}$$

$$RWR_{E,i} = \begin{cases} R_1, & \text{if} \quad T_{TL,i} \leq T_{T,i} \\ R_2, & \text{if} \quad T_{TL,i} > T_{T,i} \text{ and } Com_i = 1 \\ R_3, & \text{if} \quad T_{TL,i} > T_{T,i} \text{ and } Com_i = 0 \end{cases} \tag{9}$$

To formulate the optimization problem, (8) is converted to (10), (11), and (12) as the constraints of optimization problem.

$$RWR_{A,i} = R_1 \cdot \mu_i + R_2 \cdot (1 - \mu_i) \cdot Com_i + R_3 \cdot (1 - \mu_i) \cdot (1 - Com_i) \tag{10}$$

$$T_{RM,i} - T_{H.i} \leq M \cdot (1 - \mu_i) \tag{11}$$

$$T_{RM.i} - T_{H,i} > -M \cdot \mu_i \tag{12}$$

Similarly, (9) can be converted to (13), (14), and (15).

$$RWR_{E,i} = R_1 \cdot \upsilon_i + R_2 \cdot (1 - \upsilon_i) \cdot Com_i + R_3 \cdot (1 - \upsilon_i) \cdot (1 - Com_i) \tag{13}$$

$$T_{TL,i} - T_{T.i} \leq M \cdot (1 - \upsilon_i) \tag{14}$$

$$T_{TL.i} - T_{T,i} > -M \cdot \upsilon_i \tag{15}$$

where M is a large enough constant; and μ_i and υ_i are the auxiliary binary variables [35].

Given the discussion in Sections IV-A to IV-C, this optimization problem of minimizing total reward payment while maximizing the residents' comfort levels (thereby minimizing comfort indictor) during the summer time can be formulated as:

$$\min \sum_{i=1}^{n} RW_i + w \cdot \sum_{i=1}^{n} CI_i^2 \tag{16}$$

$s.t.$ Constraints (10), (11), (12), (13), (14), (15)

$$RW_i = PE_i \cdot (1 - SE_i) \cdot RWR_{E,i} + PA_i \cdot (1 - SA_i) \cdot RWR_{A,i} \tag{17}$$

$$TDR = \sum_{i=1}^{n} PA_i \cdot (1 - SA_i) + PE_i \cdot (1 - SE_i) \tag{18}$$

$$(1 - \delta) \cdot D \leq TDR \leq (1 + \delta) \cdot D \tag{19}$$

$$T_{T,i} = T0_{T,i} - LR_{T,i} \cdot \left(T0_{T,i} - TA_{T,i}\right) + E_i \cdot PE_i \cdot SE_i \tag{20}$$

$$T_{RM,i} = T0_{RM,i} - LR_{RM,i} \cdot \left(T0_{RM,i} - TA_{RM,i}\right) + AE_i \cdot PA_i \cdot SA_i \tag{21}$$

$$CI_i = \left| \frac{2T_{RM,i} - T_{L,i} - T_{H,i}}{T_{H,i} - T_{L,i}} \right| + \left| \frac{2T_{T,i} - T_{TL,i} - T_{TH,i}}{T_{TH,i} - T_{TL,i}} \right| \tag{22}$$

Hence, the optimization is formulated as a MIQP problem, which can be solved by available software tools.

V. Case Studies

The proposed optimal framework is performed on both a ten-resident system and a much larger system with no more than 1000 residents whose parameters are obtained from the residential energy consumption survey created by the U.S. EIA in 2009.

The first case study is designed to show how the framework schedules every appliance and rewards each resident, since more detailed information can be demonstrated in a small-scale case study. Also, this case study compares the simulation results between the optimal framework and existing DR programs. The second case study is used to show changes in residents' comfort levels and total reward costs for the LSE with different DRRs under the proposed framework.

The simulations have been carried out using General Algebraic Modeling System (GAMS), which can solve large-scale optimization problems. The MIQP problem is solved by BONMIN solver in GAMS on a desktop with Intel Xeon 3.2 GHz CPU and 8 GB RAM. Please note that, in the proposed model, only optimal control strategies over the appliances under one RLA have been considered. While there is no rigorous guarantee of the optimal solution in bounded time for the entire region, instead of an RLA service area, the following simulation results demonstrate that the proposed framework does outperform the conventional one. The performance of the proposed framework can be further improved in the future with more advanced and customized MIQP techniques.

A. Ten-Resident System

Based on the proposed framework and optimization problem formulation, several case studies have been carried out. The first test system has ten residents with the consideration of only AC units. In this system, every resident has different personal

TABLE III
TEN-RESIDENT PROFILE

ID	T_H	T_L	PA	T0	Com	AE	LR
1	75	70	1.3	72.5	0	5	0.1
2	75	70	1.4	72.5	1	5	0.1
3	75	65	1.2	70	0	5	0.3
4	80	70	1.5	75	0	5	0.2
5	75	65	1.6	70	1	6	0.3
6	75	65	1.3	70	1	5	0.1
7	75	67	1.2	71	0	4	0.1
8	77	67	1.1	70	1	4	0.2
9	77	65	1.5	71	0	5	0.2
10	75	70	1.5	72.5	1	5	0.2

TABLE IV
DRR#1 RESULT (30% AC DEMAND REDUCTION)

ID	min T_{RM} (°F)	max T_{RM} (°F)	CMFT (%)	Rate	Rewards ($)
1	70.8	74.5	100	R_1	0.38
2	70	74.3	100	R_1	0.56
3	71	72.8	100	R_1	0
4	70.1	74.1	100	R_1	0.3
5	70	70	100	R_1	0
6	66.4	70.1	100	R_1	0.78
7	70.3	72.9	100	R_1	0.24
8	70	74	100	R_1	0.22
9	67	69.8	100	R_1	0.4
10	70	74.8	100	R_1	0.3

preferences and household parameters as shown in Table III. The total demand of ACs is 13.6 kW.

Here is an example of a conventional incentive-based DR program offered in the U.S., referred to as "IDR#1" here. On hot summer days, 3 to 5 times at most per month, a typical AC will be turned off for 20 minutes. A resident who enrolls in the program will receive $8 off his/her monthly summer electricity bill as a reward. With the assumption that the power rate of the AC unit is 1.5kW, if the utility company turns off the resident's AC unit 4 times, the cost is equivalent to 33 cents/(kW·5min). Here, reward rates R_1, R_2, and R_3 in the proposed framework are roughly set to 20, 40, and 60 cents/(kW·5min), respectively. However, since the lowest reward rate ensures the residents' comfort levels and the median reward rate is higher than 33 cents/ (kW·5min), the settings of the reward rate in the optimal framework should be comparable to existing programs.

In this ten-resident case study, the RLA received two DRRs from the LSE.

1) DRR#1 With 4kW/ 20min: DRR#1 asked the RLA to reduce 4kW for 20 min among these ten residents' AC units. The results of residents' satisfaction as well as reward distributions are shown in Table IV. CMFT stands for the percentage of time when the temperature was within the comfort temperature ranges.

As for DRR#1 results, all residents were within their comfort temperature ranges. Resident #6 received the most financial rewards, due to his broad comfort temperature range and low *LR* value. A lower *LR* indicates a lower temperature change when turning off the appliances, hence a low *LR* improved capability of residents participating in DRRs. Neither resident #3 nor #5 earned rewards, because they kept their ACs on to maintain the proper room temperature due to high *LRs*. As a result, the RLA did not turn off their ACs, as long as others were able to offer enough demand reduction.

TABLE V
DRR#2 RESULT (60% AC DEMAND REDUCTION)

ID	min T_{RM} (°F)	max T_{RM} (°F)	CMFT (%)	Rate	Rewards ($)
1	70.8	74.5	100	R_1	0.58
2	70	75.8	75	R_1, R_2	1.28
3	71	72.5	100	R_1	0
4	73	79.1	100	R_1	0.6
5	70	80.2	50	R_1, R_2	1.28
6	68.8	72.8	100	R_1	0.78
7	72.2	74.6	100	R_1	0.72
8	72.6	76.1	100	R_1	0.44
9	72.8	76.3	100	R_1	0.6
10	71	76.3	75	R_1, R_2	1.2

TABLE VI
RESULT COMPARISON BETWEEN DRR#1 AND #2

	IDR#1		Optimal Framework	
	Average CMFT (%)	Equivalent Rewards ($)	Average CMFT (%)	Rewards ($)
DRR1	49.7%	5.28	100%	3.18
DRR2	46.3%	10.56	90%	7.48

2) DRR#2 With 8kW/20min: DRR#2 requests the RLA to reduce 8kW for 20 min among these ten residents' AC units. Residents' satisfaction results and reward distributions are shown in Table V.

In DRR#2, the demand to be reduced was around 60% of the total regular demand, which is tremendous. This can be traced to residents #2, #5 and #10 who bear warmer room temperature which may not be as comfortable as usual. Consequently, their financial rewards were relatively higher than others, because they were rewarded with R_2 when their room temperature went beyond their comfort temperature ranges. Note that all the three selected residents are willing to compromise.

3) Result Comparison: Table VI clearly shows that compared with the conventional incentive-based DR program IDR#1, the proposed optimal framework has the following advantages: 1) It significantly increases the resident overall comfort levels during DR events; 2) it reduces the cost for LSEs to perform DRRs; 3) The residents are rewarded for the actual contributions they made which can attract more DR program participants.

As for the optimal framework itself, the calculation time for performing both DRRs is within 0.02s, and all the appliances were fairly scheduled according to residents' preferences and parameters. Table VI shows that the increase in demands to be reduced may lead to a dramatic rise in terms of reward costs: the amount of DRR#2 was twice that of DRR#1, but the total reward cost to perform DRR#2 was about 2.34 times that of DRR#1. Because, for a relative large DRR, the RLA has to violate some residents' comfort levels to reduce enough demand. The affected will be rewarded with R_2 which increases the total reward cost.

B. Large System Test

The parameters of a large system used in this study from the RECS by U.S. EIA. The RECS data sets contain information related to appliances owned by residents, appliances'

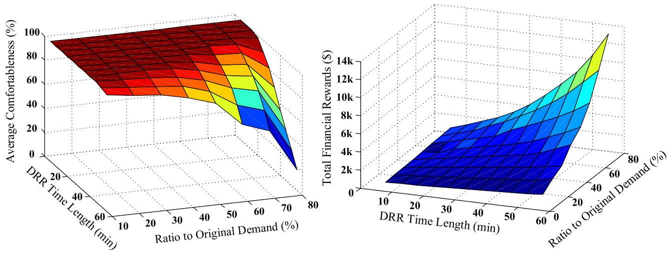
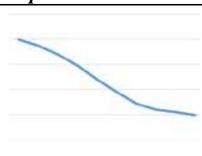
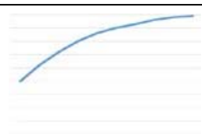
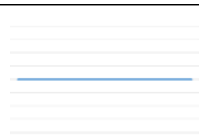


Fig. 7. The results of 500-resident system test.

TABLE VII
SCALE EFFECT COMPARISON (X-AXIS: NUMBER OF DR PROGRAM PARTICIPANTS)

	Optimal Framework	***Conventional IDR#1***
Y-Axis: Total DR Cost		
Y-Axis: Residents' Avg. CMFT		

parameters, and the common settings for those appliances. The original RECS contains the information from more than 60,000 households. In this large system study, up to 1000 residents were randomly selected, because RLA is expected to solve practical problems within this scale.

The result turns out that the optimization of 1000-resident system under the proposed optimal framework takes less than 10 seconds of computing time for each DRR.

1) The Performance of a 500-Resident System: A 500-resident system was studied with different DRRs. Fig. 7 shows the change in residents' comfort levels as well as the total reward costs for the LSE performing the DRRs with different time lengths and demand reduction amounts.

The results are reasonable as they show how, with the increase of time lengths and the amount of the demand to be reduced in DRR, the resident comfort levels dramatically fall while the total reward costs rise sharply.

2) Scale Effect: Comparing with the conventional incentive-based DR programs, such as IDR#1, the proposed optimal framework is anticipated to have better overall performance as the number of DR participants increases. With more DR program participants, the proposed framework will have more demand-side resources available for scheduling. Hence, the resident overall comfort levels can be maintained at a very high level. Consequently, in most cases, the reward rates will be R_1, and the total cost for performing DRRs will be low. However, with more program participants, the cost of IDR#1 will have a linear increase. Moreover, since resident comfort levels are not considered as the objective of IDR#1, the difference in the number of participants will not influence the average comfort levels.

In order to verify the above discussion, the simulations regarding different number of DR program participants (from 100 to 1000) were performed. The simulation results about the changing trends of cost and residents' average comfort levels regarding different numbers of participants are summarized in Table VII. It is clear that the simulation results support the statement that the proposed framework performs better with an increasing number of programs participants compared with conventional incentive-based DR programs.

VI. CONCLUSION

This paper proposes an optimal framework for aggregating residential demands. Under this framework, RLAs serve as agents of LSEs. Their role is not only to allocate DRRs among residential appliances quickly and efficiently without affecting residents' comfort levels but also to strategically reward residents for their participation, which may stimulate the potential capability of demands enrolled in incentive-based DR programs. The contributions of this work are summarized below:

1) This framework minimizes the total reward costs for LSEs to perform an efficient DRR while maintaining the comfort levels for residents.
2) A reward system is established to satisfy the needs for various types of consumers. The consumers can make a tradeoff between financial rewards and in-home comfort levels by simply submitting their energy consumption preferences over the appliances.
3) Comparing with the conventional incentive-based DR programs, the proposed framework has an economy of scale effect wherein its performance becomes better and more cost-efficient with an increasing number of DR program participants.
4) This framework can be utilized as a quick implementation for existing pilots to benefit both LSEs and residents, and further stimulates the potential capability of residential appliances enrolled in DR programs.

The future works to improve this framework include integrating the models of electrical vehicles and energy storage components, implementing new techniques of solving MIQP with limited calculation, and utilizing distributed optimization algorithms to help RLAs coordinately achieve the global optimality for the entire region.

ACKNOWLEDGEMENT

The authors would also like to thank the anonymous editor and reviewers for their comments which were very helpful for the authors to improve this paper.

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